CSE 2600
Intro. To Digital Logic & Computer Design

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Content Review

- <u>Site</u>

- Graded parts

  - Prep 1: "Due" earlier today (but still can be submitted)

  - Prep 2A: Due on next Tue, Jan 20 @ 11 am

  - Hw1: Due on next Tue, Jan 20

Review & Expansion

Chapter 1 Sections

3. **Digital Abstraction**

4. **Number Systems**
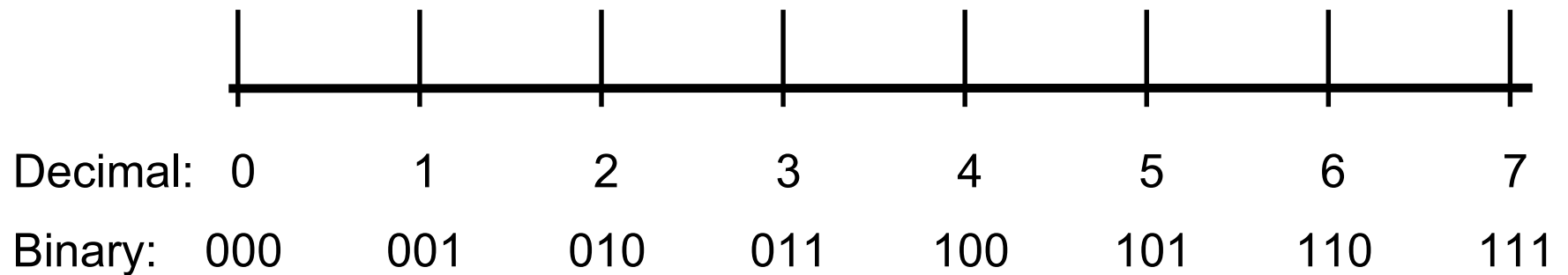
5. Logic Gates

Abstraction

- Digital discipline

  - Discrete values: 0 and 1

    - Smallest unit of information: a binary digit. Also-know-as a *Bit*

  - (Mostly) Starting at gate level

Counting

Decimal	Binary
00	000
01	001
02	010
03	011
04	100
05	101
06	110
07	111

Binary Basics: Number Line

Decimal: 0   1   2   3   4   5   6   7

Binary: 000   001   010   011   100   101   110   111

Conversions: Place Value

Place Value: Base 2
To *Decimal*
Example: 110_2 (or 3'b110)

Digits	1	1	0
Place Value (Decimal)	4	2	1
Place Value In	2^2		
Expansion	1×2^2	$+1 \times 2^1$	$+0 \times 2^0$

Digit in decimal

Value of place in decimal

Sum them all

Hexadecimal

- Convenient, compact way to deal with binary

- Each hex digit = exactly 4 binary digits

- Sixteen digits:
    0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Counting

Decimal	Binary	Hex
00	0000	
01	0001	
02	0010	
03	0011	
04	0100	
05	0101	
06	0110	
07	0111	
08	1000	
09	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	

Counting

Decimal	Binary	Hex
00	0000	0
01	0001	1
02	0010	2
03	0011	3
04	0100	4
05	0101	5
06	0110	6
07	0111	7
08	1000	8
09	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hex Notations

- Programming: 0x followed by digits, like 0xAB7

- Verilog: *n'hdigits*

Decimal number of BITS

h for Hex

Hex digits

12'hAB7

Hex to Binary

- What is 12'hAB7 in binary?

Hex to Decimal

- What is 12'hAB7 in decimal?

  - Option 1: Convert to binary and then the binary to decimal

  - Option 2: Use place-value approach

Hex to Decimal

- What is 12'hAB7 in decimal?

What is 123 in hex?

- Greedy approach

  - A lot like binary, but have to ask "how many times does this go in…"

Place Value (Decimal)	256	16	1
Place Value In terms of Base	16^2	16^1	16^0

Why hex?

- Consider 32-bit numbers

  - 0xA764 1CD8

  - Vs. 1010 0111 0110 0100 0001 1100 1101 1000

Review & Expansion: Arithmetic

Decimal Addition: Bunch of Rules
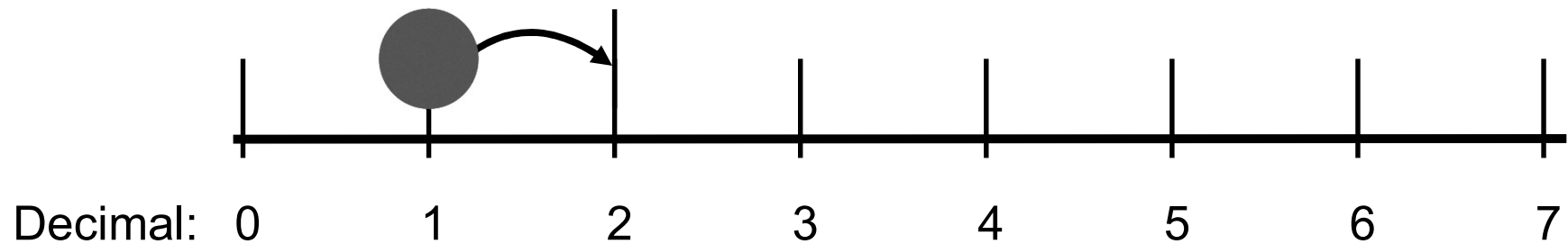
Rules just "encode" moving right on the number line

Ex: 1+2

Decimal: 0   1   2   3   4   5   6   7

Decimal Addition: Bunch of Rules

Rules just "encode" moving right on the number line

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Ex: 1+2

Decimal: 0 1 2 3 4 5 6 7
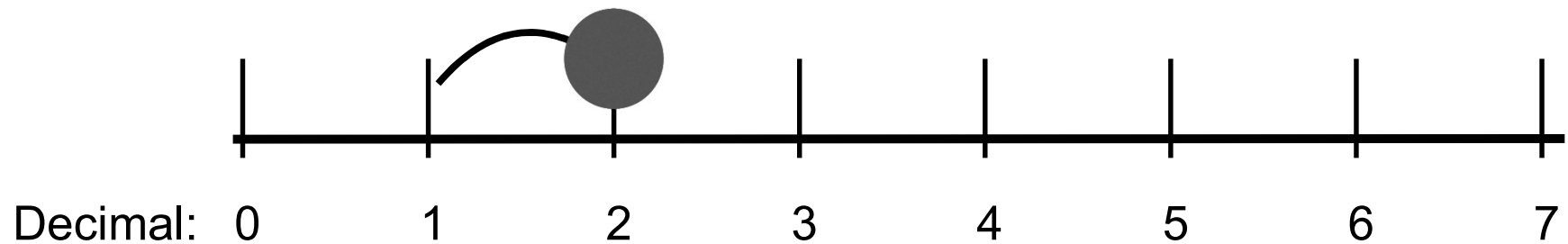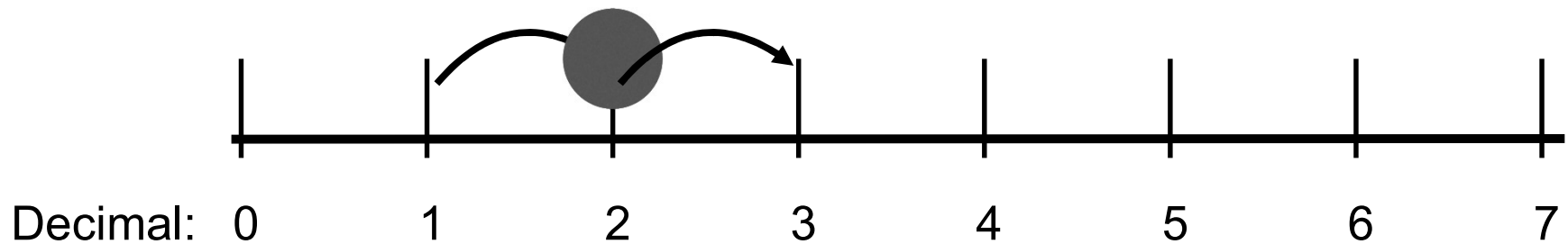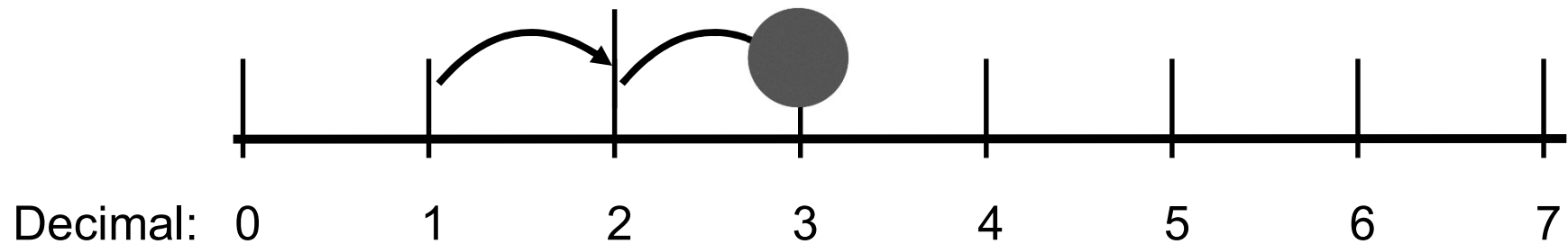
Binary Addition Rules

- Condensed

  - No ones: 0+0+0 = 00

  - One one: 0+0+1 = 01

  - Two Ones: 0+1+1 = 10

  - Three Ones: 1+1+1 = 11

Binary Addition Rules: Fully Elaborated

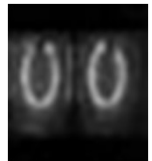|				
0+	0+	0	=	00
0+	0+	1	=	01
0+	1+	0	=	01
0+	1+	1	=	10
1+	0+	0	=	01
1+	0+	1	=	10
1+	1+	0	=	10
1+	1+	1	=	11

Review Problem

- Add 4'b0110 + 4'b0010

Fixed Width Arithmetic

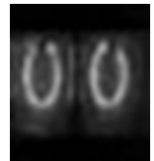# Fixed Width Arithmetic

Fixed Width Arithmetic

Fixed Width Arithmetic

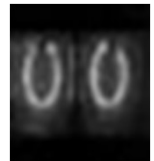- Consider the following problems:

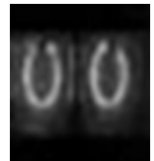 - 1 + 2 =

  - 95 + 6 =

  - 80 + 20 =

Fixed Width Arithmetic

- Consider the following problems:

  - 01 + 02 = 03 => 03

  - 95 + 06 = 101 => 01

  - 80 + 20 = 100 => 00

- Consider the following problems:

  - 03 op number = 03

  - 101 op number = 01

  - 100 op number = 00

Fixed Width Arithmetic

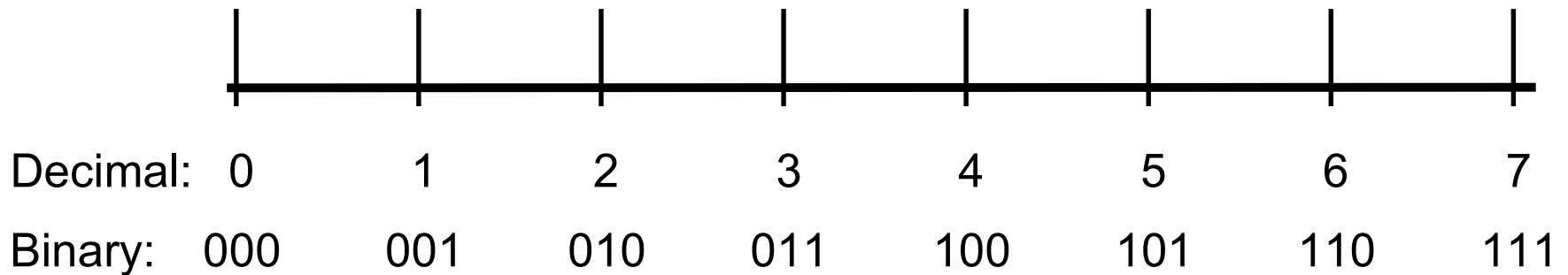- Consider the following problems:

    - 01 + 02 = 03 => 03

    - 95 + 06 = 101 => 01

    - 80 + 20 = 100 => 00

- Consider the following problems:

    - 03 % 100 = 03

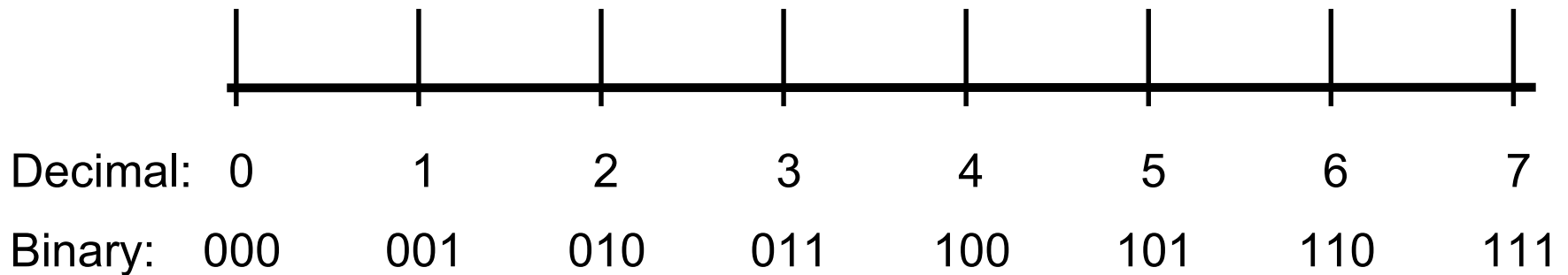   - 101 % 100 = 01

    - 100 % 100 = 00

Fixed Width Arithmetic

- $A + B$ With n-digit inputs and result in base b

- Result is $(A + B) \% b^n$

Modular Arithmetic & The Number Line (Binary, 3-bit)

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

What's 1+2?

Modular Arithmetic & The Number Line
(Binary, 3-bit)

Decimal: 0   1   2   3   4   5   6   7

Binary: 000   001   010   011   100   101   110   111

What's 6+2?

Full, binary number line (2^n values): Exceeding End Wraps!

Decimal: 0	1	2	3	4	5	6	7
Binary: 000	001	010	011	100	101	110	111
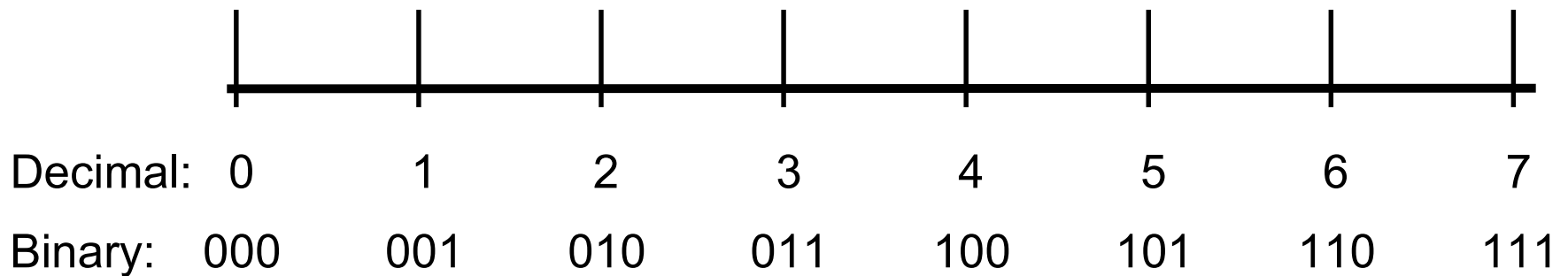
What's 6+2?

Going Negative

- What about negative numbers?

  - Easy option: Encode the concept of a sign

    - 0 = positive

    - 1 = negative

  - Pick a bit to represent the sign

Example: 4-bit sign / magnitude

- Format

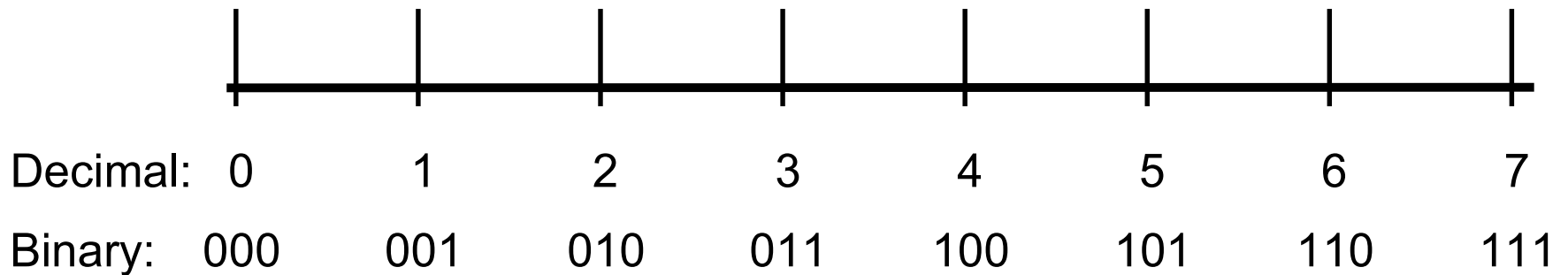| Place:	Sign	2^2	2^1	2^0

  - Sign first (common choice)

- What is the decimal value of the 4-bit, sign-magnitude

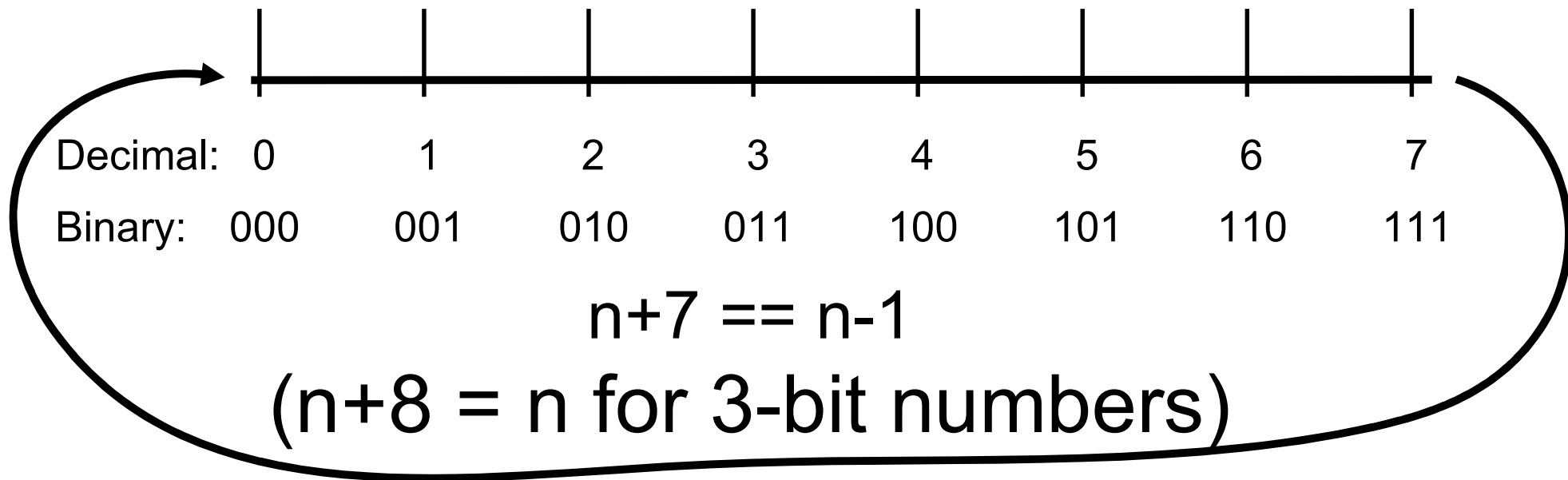 - 4'b1001

  - 4'b0110

Really Negative

- What is the decimal value of the 4-bit, sign-magnitude

    - 4'b1000

    - 4'b0000

    - Weird

- What is 4'b1001 + 4'b0110?

    - Arithmetic is messy with sign/magnitude!

UNSIGNED Again…

Challenge: Describe the result of n+7?

Decimal: 0   1   2   3   4   5   6   7

Binary: 000   001   010   011   100   101   110   111

Challenge: Describe the result of n+7?

Decimal: 0   1   2   3   4   5   6   7

Binary: 000   001   010   011   100   101   110   111

n+7 == n-1

(n+8 = n for 3-bit numbers)

Challenge: How can you emulate n-2?

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111
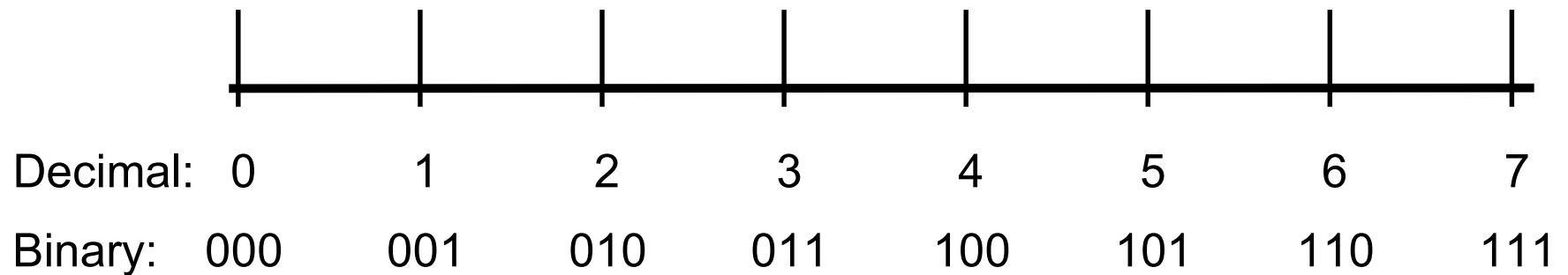
The Magic of Fixed Width numbers (modular arithmetic):
Addition can emulate subtraction!

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111
2's comp behavior:					-4	-3	-2	-1

Two's Complement Sign Representation

The "Two's Complement"

Dictionary

com·ple·ment

noun

/ˈkämpləmənt/

1. a thing that completes or brings to perfection.
   "the libretto proved a perfect **complement to** the music"

   Similar: accompaniment   companion   addition   supplement   accessory   ⌄

2. a number or quantity of something, especially that required to make a group complete.
   "at the moment we have **a full complement of** staff"

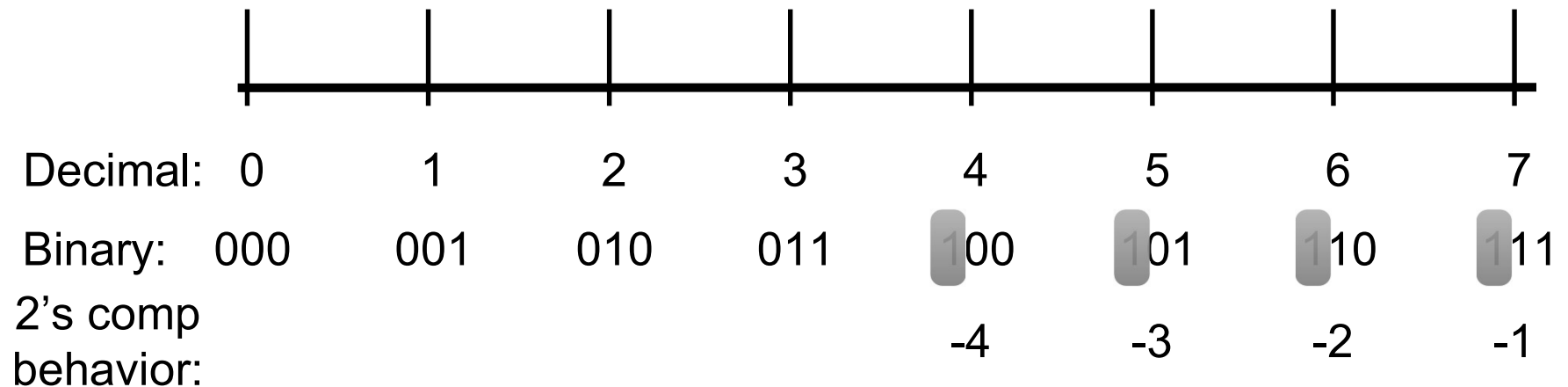   Similar: amount   total   aggregate   contingent   company   capacity   ⌄
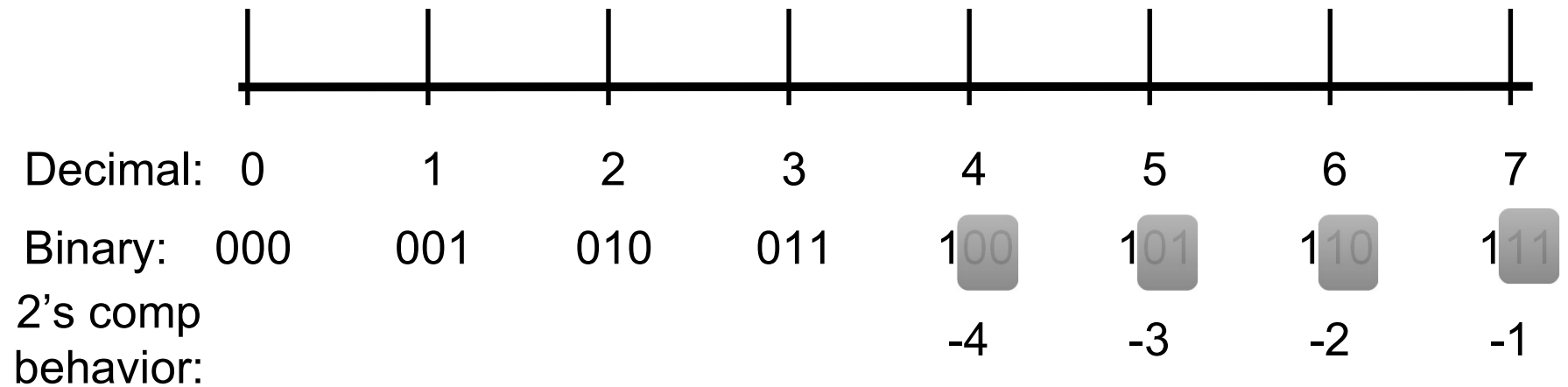
Dividing the Line

- It'll be nice if we have about half as many negatives as positives. We'll split the line in half, like we've already done.

  - How can we identify if a binary number is positive or negative?

    - Ex: 010?   Or 110?

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111
2's comp behavior:					-4	-3	-2	-1

What about conversions & place/value stuff?

Decimal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111
2's comp behavior:					-4	-3	-2	-1

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

What is the decimal value of the 3-bit, 2's complement numbers:
110
011

Consider the Upper Bit to be Negative

Place Value (Decimal)	-4	2	1
Place Value In terms of Base	$-(2^2)$	2^1	2^0

What is the 3-bit, 2's complement representation of:

2

-4

-5

4-bit Two's Complement

- What is the decimal value of the 4-bit, Two's Complement Number

  - 0xA

  - 0x7

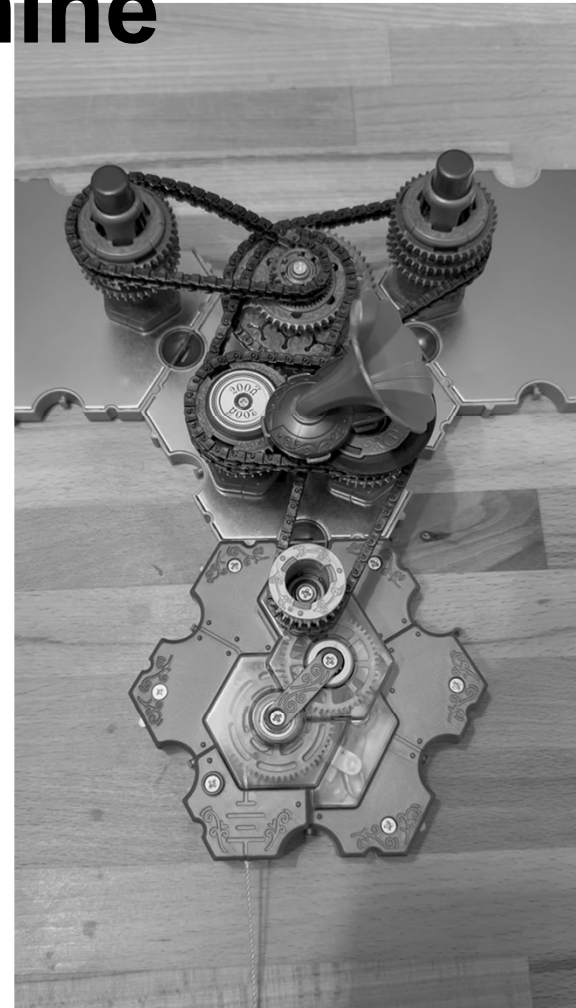# Gates & Tables

- We can describe the *final behavior* simple machines, like gates, with tables

  - Truth table

    - Row for each possible combination of inputs
      (Binary; Fixed width counting)

    - Column for each input and each output

OR Gate

- How many rows in truth table?

- How many columns?

OR: Gate…A real machine

- Spintronics game: https://upperstory.com/en/spintronics/

New Idea: Behavior OVER TIME

- Gate represents a machine of some sort machine

  - in the real world

- They are not instantaneous

The timing diagram

Intro to JLS

Coming Due Dates

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