CSE 2600
Intro. To Digital Logic & Computer Design

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Announcements

- Homework 1: Due tonight (Tuesday) at 11:59 PM.
- Homework 2: Will be posted today/tonight and due Sunday, 11:59 PM.
- Use Piazza for any immediate needs/questions. We are still determining office hours.

Review

Last Time

- Binary
- *<u>Unsigned</u>* Integers: Extension of Place-value notation used in decimal
  - Fixed width Binary (e.g., 3-bit; 4-bit; 32-bit) forms a modular ring
  - Addition rules are simple
- 2's Complement

Quiz(ish)

- Consider 5-digit integers
  - Decimal: Max value?
    (I.e., highest decimal number that can be represented)
  - Binary: Max value?
    (I.e., highest binary number that can be represented)

Chapter 2: Combinational Logic

1. Intro.
2. Boolean Equations
3. Boolean Algebra
4. From Logic to Gates

2.1 Intro: Combinational Logic

- (Purely) Combine inputs to produce outputs
    - Output depends *only* on current input, not past inputs
- Behavior of all combinational logic can be described with a table

Binary Addition Rules: Fully Elaborated

0+	0+	0	=	00
0+	0+	1	=	01
0+	1+	0	=	01
0+	1+	1	=	10
1+	0+	0	=	01
1+	0+	1	=	10
1+	1+	0	=	10
1+	1+	1	=	11

Binary Addition Rules: Inputs

Carry	A	B		Sum
0+	0+	0	=	00
0+	0+	1	=	01
0+	1+	0	=	01
0+	1+	1	=	10
1+	0+	0	=	01
1+	0+	1	=	10
1+	1+	0	=	10
1+	1+	1	=	11

Binary Addition Rules: & Outputs

Carry In	A	B		Carry Out	Sum
0+	0+	0	=	0	0
0+	0+	1	=	0	1
0+	1+	0	=	0	1
0+	1+	1	=	1	0
1+	0+	0	=	0	1
1+	0+	1	=	1	0
1+	1+	0	=	1	0
1+	1+	1	=	1	1

“Tables”

- Consider a function that has n inputs and m, 1-bit outputs
  Describe the shape / size of the complete table?

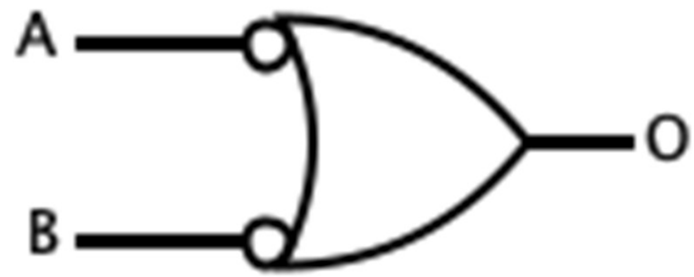
- Consider a function that has n inputs and 2, 3-bit output
  Describe the shape / size of the complete table?

Truth Tables

- Give the truth table for

Truth Tables

- Give the truth table for



2.2 Boolean Equations - History

- George: Mathematical Analysis of Logic
- Formal, algebraic approach to manipulation of binary concepts
- So?
    - Provide formal approach to manipulate concepts

2.4 Gates

- Not just electronics:
  - Scientific American, Vol. 258, No. 4 (APRIL 1988), pp. 118-121 (4 pages)
- Claude: Thesis

Boolean Algebra

Table 2.1 Axioms of Boolean algebra

	Axiom		Dual	Name
A1	$B = 0$ if $B \neq 1$	A1′	$B = 1$ if $B \neq 0$	Binary field
A2	$\overline{0} = 1$	A2′	$\overline{1} = 0$	NOT
A3	$0 \bullet 0 = 0$	A3′	$1 + 1 = 1$	AND/OR
A4	$1 \bullet 1 = 1$	A4′	$0 + 0 = 0$	AND/OR
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	A5′	$1 + 0 = 0 + 1 = 1$	AND/OR

Boolean Algebra

Table 2.2 Boolean theorems of one variable

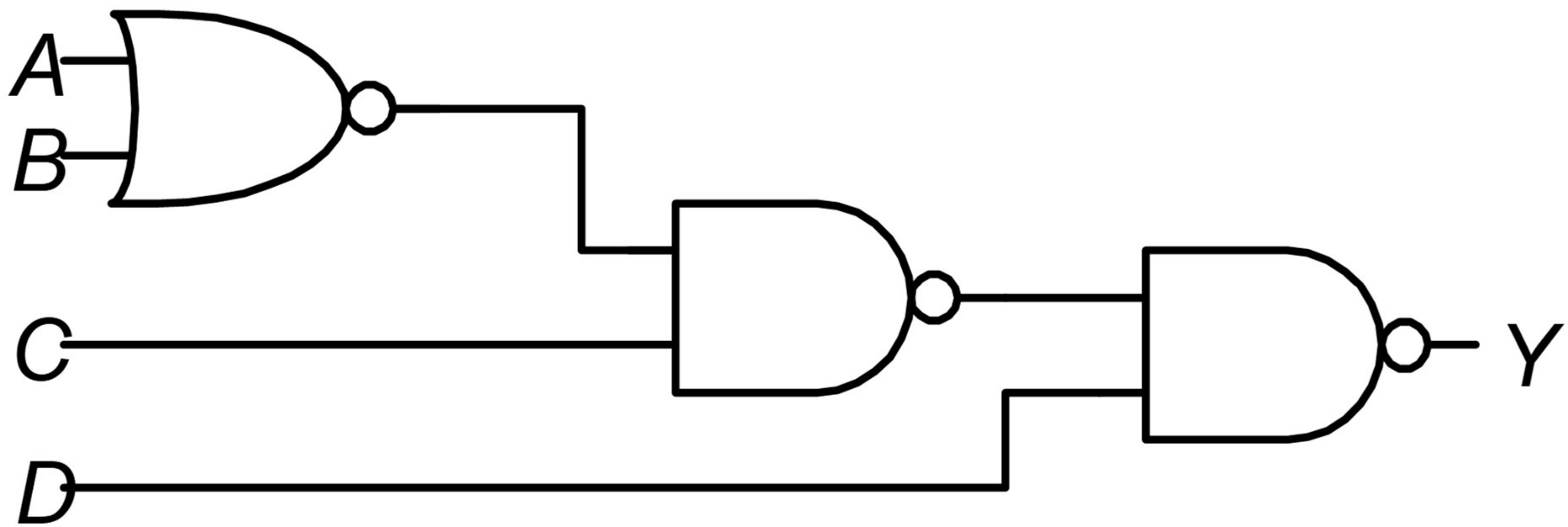
	Theorem		Dual	Name
T1	$B \bullet 1 = B$	T1′	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2′	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3′	$B + B = B$	Idempotency
T4		$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	T5′	$B + \overline{B} = 1$	Complements

Boolean Algebra

Table 2.3 Boolean theorems of several variables

	Theorem		Dual	Name
T6	$B \bullet C = C \bullet B$	T6′	$B + C = C + B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	T7′	$(B + C) + D = B + (C + D)$	Associativity
T8	$(B \bullet C) + (B \bullet D) = B \bullet (C + D)$	T8′	$(B + C) \bullet (B + D) = B + (C \bullet D)$	Distributivity
T9	$B \bullet (B + C) = B$	T9′	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	T10′	$(B + C) \bullet (B + \overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D)$ $= (B \bullet C) + (\overline{B} \bullet D)$	T11′	$(B + C) \bullet (\overline{B} + D) \bullet (C + D)$ $= (B + C) \bullet (\overline{B} + D)$	Consensus
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots}$ $= (\overline{B}_0 + \overline{B}_1 + \overline{B}_2 \ldots)$	T12′	$\overline{B_0 + B_1 + B_2 \ldots}$ $= (\overline{B}_0 \bullet \overline{B}_1 \bullet \overline{B}_2 \ldots)$	De Morgan's Theorem

Bubble Pushing



Sum of Products Form

SOP – sum-of-products

$E = \sum(\text{minterms where } E{=}1)$

C	M	E	minterm
0	0	0	$\overline{C}\ \overline{M}$
0	1	0	$\overline{C}\ M$
1	0	1	$C\ \overline{M}$
1	1	0	$C\ M$

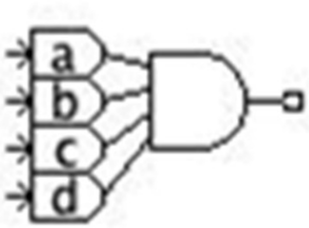
Product of Sums Form

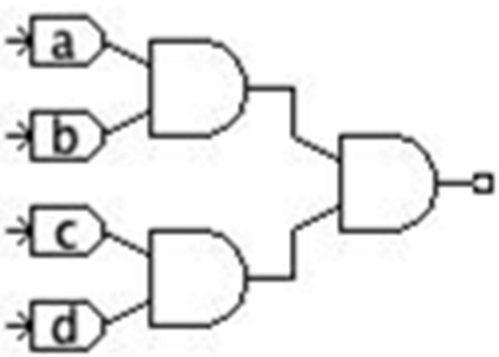
POS – product-of-sums

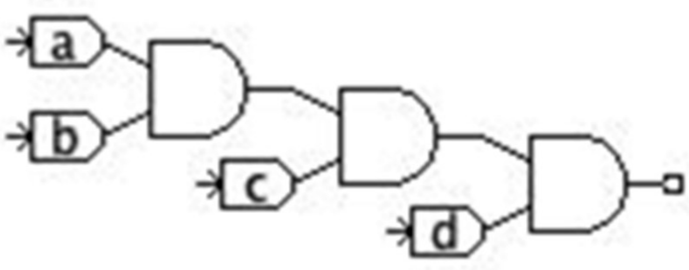
$E = \prod$(maxterms where E=0)

C	M	E	maxterm
0	0	0	$C + M$
0	1	0	$C + \overline{M}$
1	0	1	$\overline{C} + M$
1	1	0	$\overline{C} + \overline{M}$

Compare / Contrast







Combinational Logic vs. Sequential Logic

- Output of Sequential Logic
  - Depends on current inputs and *sequence* of past inputs (values and order)
  - Requires concept of memory

Exercise (putting it all together)

- Write sum-of-products form for a truth table of 3 terms (Exercise 2.1c)
- Simplify using Boolean algebra
- Draw circuit schematic
- Draw circuit in JLS
- Run logic simulation of circuit

Timing & Simulation

Next Time

- Studio
  - Prep work:
    - Install JLS
    - Check Email for attendance code
  - Check-in process (for class attendance)
    - https://washu-cse2600-sp26.github.io/studio_attendance/